LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2018**

MT 5509– ALGEBRAIC STRUCTURE - II

Date: 01-11-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART A

ANSWERALL THE QUESTIONS $(\mathbf{10 * 2 = 20} \boldsymbol{marks})$

1. Is the union of subspaces is a subspace? Justify.
2. Express the vector (1,-2,5) as a linear combination of the vectors (1,1,1),(1,2,3) and (2,-1,1) in $\mathcal{R}^3$ where $\mathcal{R}$ is the field of real numbers.
3. Define a basis of a vector space V.
4. Determine whether $T: R^2 \to R^3$ defined by $T(a,b) = (a+1, 2b, a+b)$ is a vector homomorphism or not.
5. Define orthonormal set.
6. If $T \in A(V)$ and $\lambda \in F$ and λ is an eigenvalue of T then prove that $\lambda I - T$ is singular.
7. Define trace of a matrix and give an example.
8. Show that the matrix $A = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$ is orthogonal
9. Find the rank of the following matrix over the field of rational numbers $\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$.
10. Define unitary linear transformation.

PART B

Answer any five questions: **(5x8=40 Marks)**

11. Prove that a non-empty subset W of a vector space V over F is a subspace of V if and only ifW is closed under addition and scalar multiplication.
12. If S and T are subsets of a vector space V over F, then prove the following:
    i) S is subspace of V if and only if $L(S) = S$.
    ii) $S \subseteq T$ implies that $L(S) \subseteq L(T)$.
    iii) $L\left(L(S)\right) = L(S)$.

13. If V and W are two n- dimensional vector spaces over F. Then prove that any isomorphism T of V onto W maps a basis of V onto a basis of W.

14. If $\lambda_1, \lambda_{2,} \dots \lambda_n$ are distinct eigenvalue of $T \in A(V)$ and if $v_1, v_{2,} \dots v_n$ are eigenvectors of T belonging to $\lambda_1, \lambda_{2,} \dots \lambda_n$,respectively then prove that $v_1, v_{2,} \dots v_n$are linearly independent over F.

15. Prove that $T \in A(V)$ is invertible if and only if T maps V onto V.

16. Show that any square matrix A can be expressed uniquely as the sum of a symmetric matrix and a skew-symmetric matrix.

17. For what values of λ the system of equations $x_1 + x_2 + x_3 = 1, x_1 + 2x_2 + 4x_3 = \lambda, x_1 + 4x_2 + 10x_3 = \lambda^2$ over the rational field is consistent?

18. If $T \in A(V)$is skew-Hermitian, then prove that all of its eigenvaluesare pure imaginaries.

PARTC

Answer any two questions: **(2x 20=40 Marks)**

19. Prove that the vector space V over F is a direct sum of two of its subspaces W_1 and W_2if and only if $V = W_1 + W_2$ and $W_1 \cap W_2 = \{0\}$.

20. a)If V is a vector space of finite dimension and W is subspace of V, then prove that $\dim {}^{V}/_{W} = \dim V - \dim W$.

b) If $T: U \rightarrow V$ is a homomorphism of two vector spaces over F and U has finite dimension then prove that dim $U =$ nullity of $T +$ rank of T. **(15 + 5)**

21. Prove that every finite-dimensional inner product space V has a orthonormal set as a basis.

22. a)Prove that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V onto an orthonormal basis of V.

b) If $T \in A(V)$ then prove that $T^* \in A(V)$.Also prove that

i) $(S + T)^* = S^* + T^*$

ii) $(ST)^* = T^* S^*$

iii) $(\lambda T)^* = \bar{\lambda} T^*$

iv) $(T^*)^* = T$ **(10+10)**

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